Aristotle Square in Thessaloniki, Greece

The Hanf Number for Scott Sentences of Computable Structures

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This is a joint project with Sergey Goncharov and Julia Knight.

Preliminaries

Definition

- The *Hanf number* for S is the least infinite cardinal κ such that for all $\varphi \in S$, if φ has models in all infinite cardinalities less than κ, then it has models of all infinite
- An $\mathcal{L}_{\omega_1,\omega}$-sentence ϕ *characterizes* an infinite cardinal κ, if ϕ has a model of cardinality κ, but no model of cardinality κ^+.

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Hanf Number

Theorem (Morley, López-Escobar)

Let ϕ be an $\mathcal{L}_{\omega_1,\omega}$-sentence. If ϕ has models of cardinality $\beth_\alpha$ for all $\alpha < \omega_1$, then it has models of all infinite cardinalities.

Theorem (Malitz, Baumgartner)

For every $\alpha < \omega_1$, there exists a complete $\mathcal{L}_{\omega_1,\omega}$-sentence ϕ_α that has models of size $\beth_\alpha$, but no larger.

Thus, $\beth_{\omega_1}$ is the *Hanf number* for (complete) $\mathcal{L}_{\omega_1,\omega}$-sentences.

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Main Question (Sy Friedman)

What is the Hanf number for the Scott sentences of computable structures?

Answer

Theorem (S.Goncharov, J.Knight, S.)

(a) *Let $\mathcal{A}$ be a computable structure in a computable vocabulary τ, and let ϕ be a Scott sentence for $\mathcal{A}$. If ϕ has models of cardinality $\beth_\alpha$ for all $\alpha < \omega_1^{CK}$, then it has models of all infinite cardinalities.*

(b) *There exists a partial computable function I such that for each $a \in \mathcal{O}$, $I(a)$ is a tuple of computable indices for several objects, among which are a relational vocabulary τ_a and the atomic diagram of a τ_a-structure $\mathcal{A}_a$. The Scott sentence of $\mathcal{A}_a$ characterizes the cardinal $\beth_{|a|}$, where $|a|$ is the ordinal with notation a.*

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Corollary

The Hanf number for Scott sentences of computable structures is $\beth_{\omega_1^{CK}}$.

Definition

- ω_1^{CK} is the least non-computable ordinal.
- $L_{\omega_1^{CK}}$ denotes the constructible universe at height ω_1^{CK}.
- Let τ be a computable vocabulary. A τ-structure $\mathcal{A}$ is *computable* if its atomic diagram is computable.
- An $\mathcal{L}_{\omega_1,\omega}(\tau)$-sentence is *computable* if the infinite disjunctions and conjunctions are over c.e. sets.

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Facts

- $L_{\omega_1^{CK}}$ is an admissible set.
- The subsets of ω in $L_{\omega_1^{CK}}$ are exactly the hyperarithmetical sets.
- All computable structures are elements of $L_{\omega_1^{CK}}$.
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Theorem (Morley, Barwise)

Let A be a countable admissible set with $o(A) = \gamma$, and let ϕ be a sentence of $\mathcal{L}_{\omega_1,\omega} \cap A$. Then either

- *ϕ characterizes some $\aleph_\alpha < \beth_\gamma$, or*
- *ϕ has arbitrarily large models.*

Apply this theorem for $A = L_{\omega_1^{CK}}$ and ϕ a computable $\mathcal{L}_{\omega_1,\omega}$-sentence.

Corollary

The Hanf number for computable $\mathcal{L}_{\omega_1,\omega}$-sentences is $\leq \beth_{\omega_1^{CK}}$.

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We bypass this problem by expanding the vocabulary.

Lemma

Let τ be a computable vocabulary, and let $\mathcal{A}$ be a computable τ-structure with Scott sentence ϕ. There is a computable vocabulary $\tau^* \supseteq \tau$ with a computable infinitary τ^*-sentence ϕ^* such that for any τ-structure $\mathcal{B}$,

$$\mathcal{B} \models \phi \text{ iff } \mathcal{B} \text{ has a } \tau^*\text{-expansion } \mathcal{B}^* \text{ satisfying } \phi^*.$$

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Proof: Hanf Number is $\leq \beth_{\omega_1^{CK}}$.

- From the original Scott sentence ϕ, in a computable vocabulary τ, pass to τ^* and ϕ^*.
- For each $\alpha < \omega_1^{CK}$, the sentence ϕ has a model $\mathcal{B}$ of cardinality $\beth_\alpha$. Expand these models to models of ϕ^*.
- By Morley-Barwise Theorem, ϕ^* has arbitrarily large models.
- The τ-reducts of all these models satisfy ϕ.
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For our purposes we

1. work only with relational vocabularies, but these vocabularies maybe infinite.
2. take the Fraïssé limit of some collection K of *finite* structures
3. K satisfies AP and JEP, but not HP, and
4. there are some computability assumptions on K.

The existence and uniqueness of the Fraïssé limit are rather straightforward.

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Definition (Computable representation)

Let τ be a computable relational vocabulary, and let K be a (countable) family of finite τ-structures. A *computable representation* of K is a computable sequence $\mathbb{K}$, with $\mathbb{K}(i) = (e_i, n_i)$ such that

1. φ_{e_i} is the characteristic function of the atomic diagram of a structure C_i isomorphic to some element of K, and D_{n_i} is the universe of C_i,
2. for each $M \in K$, there is some i such that $C_i \cong M$.

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Definition (Strong Embedding Property)

Let τ be a computable relational vocabulary, and let K be a family of finite τ-structures. Suppose that $(C_i)_{i\in\omega}$ is the sequence of structures given by a computable representation $\mathbb{K}$ of K.

1. The corresponding *embedding relation*, denoted by $E(\mathbb{K})$, is the set of triples (i, j, f) such that f is an embedding of C_i into C_j.
2. We say that $\mathbb{K}$ has the *strong embedding property* if $E(\mathbb{K})$ is computable.

If τ is finite, $E(\mathbb{K})$ is computable.
If τ is infinite, this need not be the case.

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Theorem

There is a computable vocabulary τ and a family $\boldsymbol{K}$ of finite τ-structures that has a computable representation $\mathbb{K}$ of such that $E(\mathbb{K})$ is not even c.e.

Definition

Let

- τ be a computable relational vocabulary,
- K a family of finite τ-structures,
- $\mathbb{K}$ a computable representation of K with $(C_i)_{i\in\omega}$ the corresponding sequence of structures in $\mathbb{K}$ and
- $\mathcal{A}$ be a Fraïssé limit of K.

Denote by $E(\mathbb{K}, \mathcal{A})$ the set of pairs (i, f) such that f is an embedding of C_i into $\mathcal{A}$.

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Theorem (Computable Fraïssé Limit)

Let

- τ *be a computable relational vocabulary,*
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Then there is a computable Fraïssé limit $\mathcal{A}$ such that $E(\mathbb{K}, \mathcal{A})$ is computable.
In fact, we have a uniform effective procedure for passing from τ, $\mathbb{K}$ and $E(\mathbb{K})$ to $D(\mathcal{A})$ and $E(\mathbb{K}, \mathcal{A})$.

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Construction

The construction in the second half of the theorem is based on the following idea.

For any triple of distinct elements $v, u \in P(\kappa)$, let

$$F(v, u) = \text{ least } \alpha \in \kappa \text{ such that } v(\alpha) \neq u(\alpha).$$

For all $v_0, v_1, v_2 \in P(\kappa)$,

- if $F(v_0, v_1) \neq F(v_0, v_2)$, then $F(v_1, v_2) = \min\{F(v_0, v_1), F(v_0, v_2)\}$.
- if $F(v_0, v_1) = F(v_0, v_2)$, then $F(v_1, v_2) > F(v_0, v_1)$.

Call this last property $\bigstar$.

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Call this last property $\bigstar$.

Building on this idea consider the collection K of finite structures that satisfy the following:

1. V, M, U partition the universe
2. M is linearly ordered by $<$.
3. There is a function F from $[V]^2$ to M.
4. F satisfies $\bigstar$.
5. U is linearly ordered by $<'$.
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Theorem

1. K satisfies AP and JEP, and therefore has a Fraïssé limit $\mathcal{A}$.
2. If ϕ is the Scott sentence of $\mathcal{A}$, then in all models of ϕ, $|U| \leq |V| \leq 2^{|M|}$.
3. If $(L, \prec)$ is a dense linear order with a cofinal sequence of order type κ, then there is a model of ϕ with $(M, <) \cong (L, \prec)$ and V, U both have size 2^{κ}.

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Based on this idea we build by computable transfinite induction on ordinal notations $a \in \mathcal{O}$ the following function I.

For every a, $I(a)$ is a tuple of computable indices including the following:

1. some vocabulary τ_a
2. a computable representation $\mathbb{K}_a$ for some collection K_a of finite τ_a-structures
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Moreover, the Scott sentence ϕ_a of $\mathcal{A}_a$ characterizes the cardinal $\beth_{|a|}$, where $|a|$ is the ordinal with notation a.

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